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MA 425/525
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Homework 4

Due October 2nd on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

There are some hints on the next page.

Do 2.1.18, 2.1.20ac from pages 85 and 86, and 2.2.1, 2.2.8, 2.2.10 from page 103. For these last two, give the answer both using summation notation and by writing out the first four nonzero terms and an ellipsis as in

$$ze^{z^2} = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{n!} = z + z^3 + \frac{z^5}{2} + \frac{z^7}{6} + \cdots$$

Also do the following problems:

1. Show that if Γ is a contour from p to q and f is an analytic function, then $\int_\Gamma f'(z)\,dz = f(q) - f(p)$.
2. Use the result of problem 1 to evaluate $\int_\Gamma e^{2z}\,dz$, where Γ is the circular arc starting at 1, passing through $2+i$, and ending at 0.
3. Let $f_n(z) = z^{23}\operatorname{Log}(1 - z^n)$. Find $f_n^{(30)}(0)$ for each positive integer n.
4. Let $f(z) = \sum_{n=1}^{\infty} nz^n$. For which real values of θ is $f(1 + e^{i\theta})$ defined? For those values of θ, find a concise formula for $f(1 + e^{i\theta})$.

Hints:

For 2.1.20ac, solve the Cauchy–Riemann equations (from Theorem 1 on page 80) for v; doing this is a version of finding a potential function given a conservative vector field: see Example 2b of `https://tutorial.math.lamar.edu/Classes/CalcIII/ConservativeVectorField.aspx` for a refresher if needed.

For the last four problems:

1. Use parametrization and the line integral definition (at the top of page 60) followed by the chain rule and the fundamental theorem of calculus.
2. Using the result of 1 makes it unnecessary to parametrize Γ.
3. It may be helpful to look at $n = 1$, 2, etc. individually until you see the pattern.
4. Find and simplify an antiderivative for $f(z)/z$.