

### Homework 4

Due October 4th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is <https://archive.org/details/complex-variables-2ed-dover-page/n23/mode/2up>.

- Section 2.1 (pages 84 and 85) # 6, 16.
- Section 2.2 (page 103) # 1, 8, 10. For these last two, give the answer both using summation notation and by writing out the first four nonzero terms and an ellipsis as in

$$ze^{z^2} = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{n!} = z + z^3 + \frac{z^5}{2} + \frac{z^7}{6} + \dots$$

- Nonbook problems:
  1. Let  $f_n(z) = z^{23} \operatorname{Log}(1 - z^n)$ . Find  $f_n^{(30)}(0)$  for each positive integer  $n$ .
  2. Let  $f$  and  $g$  be two complex functions related by  $g(z) = \overline{f(\bar{z})}$ . Prove that  $g$  is analytic if and only if  $f$  is.
  3. Let  $f(z) = \sum_{n=1}^{\infty} nz^n$ . For which real values of  $\theta$  is  $f(1 + e^{i\theta})$  defined? For those values of  $\theta$ , find a concise formula for  $f(1 + e^{i\theta})$ .

*Hint:* Find an antiderivative of  $f(z)/z$ .